

Softly broken lepton number $L_e - L_\mu - L_\tau$ with non-maximal solar neutrino mixing

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Abstract

We consider the most general neutrino mass matrix which leads to $\theta_{13} = 0$, and present the formulae needed for obtaining the neutrino masses and mixing parameters in that case. We apply this formalism to a model based on the lepton number $\bar{L} = L_e - L_\mu - L_\tau$ and on the seesaw mechanism. This model needs only one Higgs doublet and has only two right-handed neutrino singlets. Soft \bar{L} breaking is accomplished by the Majorana mass terms of the right-handed neutrinos; if the \bar{L} -conserving and \bar{L} -breaking mass terms are of the same order of magnitude, then it is possible to obtain a consistent \bar{L} model with a solar mixing angle significantly smaller than 45° . We show that the predictions of this model, $m_3 = 0$ and $\theta_{13} = 0$, are invariant under the renormalization-group running of the neutrino mass matrix.

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1 Introduction

In recent times there has been enormous experimental progress in our knowledge of the mass-squared differences and of the mixing of light neutrinos—for a review see, for instance, [1]. Unfortunately, to this progress on the experimental and phenomenological—i.e. neutrino oscillations [2] and the MSW effect [3]—fronts there has hardly been a counterpart in our theoretical understanding of neutrino masses and lepton mixing—for a review see, for instance, [4].

It has been conclusively shown that the lepton mixing matrix is substantially different from the quark mixing matrix. Whereas the solar mixing angle, θ_{12} , is large— $\theta_{12} \sim 33^\circ$ —and the atmospheric mixing angle, θ_{23} , could even be maximal, the third mixing angle, θ_{13} , is small—there is only an upper bound on it which, according to [1], is given at the 3σ level by $\sin^2 \theta_{13} < 0.047$. The true magnitude of θ_{13} will be crucial in the future experimental exploration of lepton mixing, and it is also important for our theoretical understanding of that mixing—see, for instance, [5].

In this letter we contemplate the possibility that at some energy scale a flavour symmetry exists such that θ_{13} is exactly zero. It has been shown [6] that, in the basis where the charged-lepton mass matrix is diagonal, the most general neutrino mass matrix which yields $\theta_{13} = 0$ is given, apart from a trivial phase convention [6], by

$$\mathcal{M}_\nu = \begin{pmatrix} X & \sqrt{2}A \cos(\gamma/2) & \sqrt{2}A \sin(\gamma/2) \\ \sqrt{2}A \cos(\gamma/2) & B + C \cos \gamma & C \sin \gamma \\ \sqrt{2}A \sin(\gamma/2) & C \sin \gamma & B - C \cos \gamma \end{pmatrix}, \quad (1)$$

with parameters X , A , B , and C which are in general complex. The mass matrix (1), but not necessarily the full Lagrangian, enjoys a \mathbb{Z}_2 symmetry [6, 7]—see also [8]—defined by

$$S(\gamma) \mathcal{M}_\nu S(\gamma) = \mathcal{M}_\nu, \quad (2)$$

with an orthogonal 3×3 matrix

$$S(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & \sin \gamma & -\cos \gamma \end{pmatrix} \quad (3)$$

which satisfies $S(\gamma) = [S(\gamma)]^T$ and $[S(\gamma)]^2 = \mathbb{1}$. We may remove two unphysical phases from (1), e.g. by choosing X and A to be real, and then there are seven real parameters in that mass matrix. Those seven parameters correspond to the following seven physical quantities: the three neutrino masses $m_{1,2,3}$, the solar and atmospheric mixing angles, and two Majorana phases. The only prediction of the mass matrix (1) is $\theta_{13} = 0$; however, that prediction entails the non-observability of the Dirac phase in lepton mixing—in general there are *nine* physical quantities in the neutrino masses and mixings.

Expressed in terms of the parameters¹ of \mathcal{M}_ν , one obtains masses given by [6, 10]

$$m_3 = |B - C| \quad (4)$$

¹The procedure for obtaining the neutrino masses and the lepton mixing matrix from the parameters of a fully general neutrino mass matrix has been given in [9].

and

$$m_{1,2}^2 = \frac{1}{2} \left[|X|^2 + |D|^2 + 4|A|^2 \mp \sqrt{(|X|^2 + |D|^2 + 4|A|^2)^2 - 4|XD - 2A^2|^2} \right], \quad (5)$$

with

$$D \equiv B + C; \quad (6)$$

while the mixing angles are given by

$$\theta_{23} = |\gamma/2| \quad (7)$$

and

$$(m_2^2 - m_1^2) \sin 2\theta_{12} = 2\sqrt{2} |X^*A + A^*D|, \quad (8)$$

$$(m_2^2 - m_1^2) \cos 2\theta_{12} = |D|^2 - |X|^2. \quad (9)$$

The only Majorana phase which—for $\theta_{13} = 0$ —plays a role in neutrinoless $\beta\beta$ decay is the phase $\Delta = \arg[(U_{e2}/U_{e1})^2]$, where U_{e1} and U_{e2} are matrix elements of the lepton mixing (PMNS [2, 11]) matrix U . The phase Δ is given by [12]

$$8 \operatorname{Im}(X^*D^*A^2) = m_1 m_2 (m_1^2 - m_2^2) \sin^2 2\theta_{12} \sin \Delta. \quad (10)$$

The other Majorana phase is practically unobservable [13].²

In specific models with $\theta_{13} = 0$, the neutrino mass matrix (1) is further restricted:

- The \mathbb{Z}_2 model of [10], which is based on the non-Abelian group $O(2)$ [14], yields maximal atmospheric neutrino mixing, i.e. $\gamma = \pi/2$, and has six physical parameters.
- The D_4 model of [15], which is based on the discrete group D_4 , also has $\gamma = \pi/2$ and, in addition, it predicts $XC = A^2$. The number of parameters is four—in that model the Majorana phases are expressible in terms of the neutrino masses and of the solar mixing angle.
- The softly-broken- D_4 model of [7] is a generalization of the D_4 model: the atmospheric mixing angle is undetermined, but $XC = A^2$ still holds.
- The seesaw model of the first line of Table I of [16], which is based on the Abelian group \mathbb{Z}_4 , reproduces matrix (1) without restrictions.

In this letter we consider the $U(1)$ symmetry generated by the lepton number $\bar{L} \equiv L_e - L_\mu - L_\tau$ [17]. It is well known that exact \bar{L} symmetry enforces $\theta_{13} = 0$ (with $X = B = C = 0$ in (1)), while an approximate \bar{L} symmetry tends to produce either a solar mixing angle too close to 45° or a solar mass-squared difference too close to the atmospheric mass-squared difference [18]. A possible way out of this dilemma is to assume a significant contribution to U from the diagonalization of the charged-lepton mass matrix [19]; another possibility is a significant breaking of \bar{L} [20]. Here we discuss a \bar{L}

²When $|X| = |D|$ and $X^*A = -A^*D$ one should use, instead of (5) and (8)–(10), $m_1 = m_2 = \sqrt{|X|^2 + 2|A|^2}$, $\Delta = \pi$, and $\cos 2\theta_{12} = |X|/m_1$.

model, first proposed in [21], which makes use of the seesaw mechanism [22] with *only two* right-handed neutrino singlets ν_R . The $U(1)_{\bar{L}}$ symmetry is softly broken in the Majorana mass matrix of the ν_R , but—contrary to what was done in [21]—the soft breaking is assumed here to be rather ‘strong’, in order to achieve a solar mixing angle significantly smaller than 45° . The model presented in Section 2 predicts a mass matrix (1) with $B = C$, i.e. it predicts $m_3 = 0$ together with $\theta_{13} = 0$. We will show in Section 3 that these predictions *are stable* under the renormalization-group running from the seesaw scale down to the electroweak scale. Next, we show in Section 4 that our model does *not* provide enough leptogenesis to account for the observed baryon-to-photon ratio of the Universe. We end in Section 5 with our conclusions.

2 The model

The lepton number $\bar{L} = L_e - L_\mu - L_\tau$ has a long history in model building [17, 18]. In this letter we rediscuss the model of [21], which has *only one Higgs doublet*, ϕ , and two right-handed neutrinos, ν_{R1} and ν_{R2} , with the following assignments of the quantum number \bar{L} :

$$\begin{array}{c|cccccc} & \nu_e, e & \nu_\mu, \mu & \nu_\tau, \tau & \nu_{R1} & \nu_{R2} & \phi \\ \hline \bar{L} & 1 & -1 & -1 & 1 & -1 & 0 \end{array}. \quad (11)$$

This model is a simple extension of the Standard Model which incorporates the seesaw mechanism [22]. The right-handed neutrino singlets have a Majorana mass term

$$\mathcal{L}_M = -\frac{1}{2} \bar{\nu}_R M_R C \bar{\nu}_R^T + \text{H.c.}, \quad (12)$$

(where C is the charge-conjugation matrix in spinor space) with

$$M_R = \begin{pmatrix} R & M \\ M & S \end{pmatrix}. \quad (13)$$

The elements of the matrix M_R are of the heavy seesaw scale. The entry M in M_R is compatible with \bar{L} symmetry, while the entries R and S break that lepton number softly. The breaking of \bar{L} is soft since the Majorana mass terms have dimension three. Because of the $U(1)$ symmetry associated with \bar{L} , the neutrino Dirac mass matrix has the structure [21]³

$$M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & b' & b'' \end{pmatrix}. \quad (14)$$

Then the effective Majorana mass matrix of the light neutrinos is given by the seesaw formula

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D = \frac{1}{M^2 - RS} \begin{pmatrix} Sa^2 & -Mab' & -Mab'' \\ -Mab' & Rb'^2 & Rb'b'' \\ -Mab'' & Rb'b'' & Rb''^2 \end{pmatrix}. \quad (15)$$

³We are assuming, without loss of generality, the charged-lepton mass matrix to be already diagonal.

In the case of \bar{L} conservation, i.e. when $R = S = 0$, we have $m_1 = m_2$ and θ_{12} is 45° ; this is a well known fact. Non-zero mass parameters R and S induce $\Delta m_\odot^2 \equiv m_2^2 - m_1^2 \neq 0$ and allow a non-maximal solar mixing angle.⁴ The phases of a , b' , and b'' are unphysical; in the following we shall assume those parameters to be real and positive. The only physical phase is [21]

$$\alpha \equiv \arg(R^* S^* M^2). \quad (16)$$

CP conservation is equivalent to α being a multiple of π . Defining $d \equiv M^2 - RS$ and $b \equiv \sqrt{b'^2 + b''^2}$, we see that (15) has the form (1) with

$$X = \frac{Sa^2}{d}, \quad A = -\frac{Mab}{\sqrt{2}d}, \quad B = C = \frac{Rb^2}{2d}, \quad (17)$$

and

$$\cos \frac{\gamma}{2} = \frac{b'}{b}, \quad \sin \frac{\gamma}{2} = \frac{b''}{b}. \quad (18)$$

As advertised in the introduction, $C = B$ and therefore $m_3 = 0$, while X , A , and B are independent parameters. Since $m_3 = 0$ the neutrino mass spectrum displays inverted hierarchy. The present model has five real parameters— $|X|$, $|A|$, $|B|$, γ , and α —which correspond to the physical observables $m_{1,2}$, θ_{12} , θ_{23} , and the Majorana phase Δ (the second Majorana phase is unphysical in this case because $m_3 = 0$).

Let us now perform a consistency check by using all the available input from the neutrino sector. We have the following observables at our disposal: the effective Majorana mass in neutrinoless $\beta\beta$ decay $m_{\beta\beta}$, the solar mass-squared difference Δm_\odot^2 , the atmospheric mass-squared difference Δm_{atm}^2 , the solar mixing angle θ_{12} , and the atmospheric mixing angle θ_{23} . In a three-neutrino scenario the definition of Δm_{atm}^2 is not unique; we define $\bar{m}^2 \equiv (m_1^2 + m_2^2)/2$ and use $\Delta m_{\text{atm}}^2 \simeq \bar{m}^2$, which is valid in this model because of the inverted mass hierarchy. The relation $\gamma = 2\theta_{23}$ plays no role in the following discussion, which consists in determining the four parameters $|X|$, $|A|$, $|D| = 2|B|$, and α as functions of the four observables $m_{\beta\beta}$, Δm_\odot^2 , Δm_{atm}^2 , and θ_{12} . We note that, because of (16) and (17), $\alpha = \arg(D^* X^* A^2)$.

We first note that, in (1), $m_{\beta\beta}$ is just given by $|X|$:

$$|X| = m_{\beta\beta}. \quad (19)$$

We then use (9) to write

$$|D|^2 = m_{\beta\beta}^2 + \Delta m_\odot^2 \cos 2\theta_{12}. \quad (20)$$

From (5), $\bar{m}^2 = 2|A|^2 + (|X|^2 + |D|^2)/2$, hence

$$|A|^2 = \frac{1}{2} \left(\bar{m}^2 - m_{\beta\beta}^2 - \frac{1}{2} \Delta m_\odot^2 \cos 2\theta_{12} \right). \quad (21)$$

Since $|A| \geq 0$ we have the bound

$$m_{\beta\beta}^2 \leq \bar{m}^2 - \frac{1}{2} \Delta m_\odot^2 \cos 2\theta_{12}. \quad (22)$$

⁴The case of non-zero R and S has also been considered in [23].

In order to find α we start from (8), writing

$$(\Delta m_\odot^2)^2 \sin^2 2\theta_{12} = 8 |A|^2 (|X|^2 + |D|^2 + 2 |X| |D| \cos \alpha). \quad (23)$$

We define

$$p \equiv \frac{2m_{\beta\beta}^2}{\Delta m_\odot^2 \cos 2\theta_{12}}, \quad (24)$$

$$\rho \equiv \frac{2\bar{m}^2}{\Delta m_\odot^2 \cos 2\theta_{12}}, \quad (25)$$

and obtain

$$\cos \alpha = \left[-(1+p) + \frac{\tan^2 2\theta_{12}}{2(\rho-1-p)} \right] \frac{1}{\sqrt{p(p+2)}}. \quad (26)$$

Thus we have expressed all the parameters of the model in terms of physical quantities.

The parameter ρ is known and it is quite large: using the mean values of the mixing parameters [1] $\Delta m_\odot^2 \sim 8.1 \times 10^{-5} \text{ eV}^2$, $\Delta m_{\text{atm}}^2 \sim 2.2 \times 10^{-3} \text{ eV}^2$, and $\sin^2 \theta_{12} \sim 0.30$, we find $\rho \sim 136$. The parameter p , on the other hand, is unknown. Equation (26) requires that a non-zero range $[p_-, p_+]$ for p exists for which the right-hand side of that equation lies in between -1 and $+1$. One finds that

$$p_\pm = \frac{\rho}{2} \left(1 + \cos^2 2\theta_{12} \pm \sin^2 2\theta_{12} \sqrt{1 - \frac{1}{\rho^2 \cos^2 2\theta_{12}}} \right) - 1. \quad (27)$$

Since ρ is large this can be approximated by

$$p_- \simeq \rho \cos^2 2\theta_{12} - 1, \quad p_+ \simeq \rho - 1, \quad (28)$$

or

$$\bar{m}^2 \cos^2 2\theta_{12} - \frac{\Delta m_\odot^2 \cos 2\theta_{12}}{2} \lesssim m_{\beta\beta}^2 \lesssim \bar{m}^2 - \frac{\Delta m_\odot^2 \cos 2\theta_{12}}{2}. \quad (29)$$

In this approximation, the upper bound on $m_{\beta\beta}$ coincides with the one in (22). With good accuracy we have in this model

$$\sqrt{\Delta m_{\text{atm}}^2 \cos 2\theta_{12}} \lesssim m_{\beta\beta} \lesssim \sqrt{\Delta m_{\text{atm}}^2}. \quad (30)$$

This is one of the predictions of the model. Thus, if the claim $m_{\beta\beta} > 0.1 \text{ eV}$ of [24] is confirmed, then the present model will be ruled out since $\sqrt{\Delta m_{\text{atm}}^2} \sim 0.047 \text{ eV}$.

From (8), (9), and (17) we find the following expression for the solar mixing angle:

$$\tan 2\theta_{12} = \frac{2 |M| ab}{|R| b^2 - |S| a^2} \frac{||R| b^2 + |S| a^2 e^{i\alpha}|}{|R| b^2 + |S| a^2}. \quad (31)$$

This equation shows that non-maximal solar neutrino mixing is easily achievable when $|R|$ and $|S|$ are of the same order of magnitude as $|M|$. This is what we mean with ‘strong’

soft \bar{L} breaking, namely that the Majorana mass terms which violate \bar{L} softly (i.e. R and S) and the one which conserves \bar{L} (i.e. M) are of the same order of magnitude.⁵

One may ask whether it is possible to evade this feature and assume $|R|, |S| \ll |M|$. In that case, since experimentally $\tan 2\theta_{12} \simeq 2.3$, and since the second fraction in the right-hand side of (31) cannot be larger than 1, we would conclude that $b/a \sim |M/R|$. But then $|R|b^2$ would be much larger than $|S|a^2$ and therefore $|D| \gg |X|$ which, from (19) and (20), means that $m_{\beta\beta}^2 \ll \Delta m_\odot^2 \cos 2\theta_{12}$. This contradicts our previous finding that $m_{\beta\beta}^2$ must be of the order of magnitude of Δm_{atm}^2 . We thus conclude that the hypothesis $|R|, |S| \ll |M|$ is incompatible with the experimental data.

3 Radiative corrections

We have not yet taken into account the fact that the energy scale where \bar{L} -invariance holds and the mass matrices M_D and \mathcal{M}_ν have the forms (14) and (15), respectively, is the seesaw scale. Since our model has *only one* Higgs doublet, the relation between the mass matrix $\mathcal{M}_\nu^{(0)}$ at the seesaw scale and the mass matrix \mathcal{M}_ν at the electroweak scale is simply given by [25]

$$\mathcal{M}_\nu = I \mathcal{M}_\nu^{(0)} I, \quad (32)$$

where I is a diagonal, positive, and non-singular matrix, since the charged-lepton mass matrix is diagonal. Now, suppose there is a vector $u^{(0)}$ such that $\mathcal{M}_\nu^{(0)} u^{(0)} = 0$. Then the vector $u \equiv I^{-1} u^{(0)}$ is an eigenvector to \mathcal{M}_ν with eigenvalue zero.⁶ Moreover, if one entry of $u^{(0)}$ is zero, then the corresponding entry of u is zero as well, due to I being diagonal. We stress that these observations only hold for eigenvectors with eigenvalue zero.

Applying this to the present model, we find that $m_3 = 0$ together with $\theta_{13} = 0$ are predictions *stable under the renormalization-group evolution*. The matrices \mathcal{M}_ν and $\mathcal{M}_\nu^{(0)}$ are related through $M_D = M_D^{(0)} I$, where $M_D^{(0)}$ is the neutrino Dirac mass matrix at the seesaw scale; again, due to I being *diagonal*, both Dirac mass matrices have the same form (14). Therefore, all our discussions in the previous section hold for the physical quantities at the low (electroweak) scale.

4 Leptogenesis

The model in this letter has very few parameters and only one Higgs doublet. Therefore, it allows clear-cut predictions for leptogenesis—for reviews see, for instance, [26]. It turns out that the computations for this model resemble closely the ones for the \mathbb{Z}_2 model [10], which were performed in a previous paper [12]. We give here only the gist of the argument.

Let the matrix M_R in (13) be diagonalized by the 2×2 unitary matrix

$$V = \begin{pmatrix} c' e^{i\omega} & s' e^{i\sigma} \\ -s' e^{i\tau} & c' e^{i(\sigma+\tau-\omega)} \end{pmatrix} \quad (33)$$

⁵In [21] we assumed $|R|, |S| \ll |M|$ and ended up with almost-maximal solar mixing, which was still allowed by the data at that time.

⁶This statement would still be true for a non-diagonal matrix I .

as

$$V^T M_R V = \text{diag}(M_1, M_2), \quad (34)$$

with real, non-negative M_1 and M_2 . We assume $M_1 \ll M_2$. In (33), $c' \equiv \cos \theta'$ and $s' \equiv \sin \theta'$, where θ' is an angle of the first quadrant. Defining the Hermitian matrix

$$H \equiv V^T M_D M_D^\dagger V^*, \quad (35)$$

the relevant quantity for leptogenesis is [26]

$$\mathcal{Q} \equiv \frac{\text{Im}[(H_{12})^2]}{(H_{11})^2}. \quad (36)$$

One may use as input for leptogenesis the heavy-neutrino masses $M_{1,2}$ together with $m_{1,2}$, θ_{12} , and the Majorana phase Δ . One can demonstrate that a and $b = \sqrt{b'^2 + b''^2}$ satisfy

$$a^2 b^2 = m_1 m_2 M_1 M_2 \quad (37)$$

and

$$\begin{aligned} & \left| s_{12}^2 m_1 + c_{12}^2 m_2 e^{i\Delta} \right|^2 a^4 + \left| c_{12}^2 m_1 + s_{12}^2 m_2 e^{i\Delta} \right|^2 b^4 \\ &= m_1^2 m_2^2 (M_1^2 + M_2^2) - 2m_1 m_2 M_1 M_2 c_{12}^2 s_{12}^2 \left| m_1 - m_2 e^{i\Delta} \right|^2, \end{aligned} \quad (38)$$

where $c_{12} \equiv \cos \theta_{12}$ and $s_{12} \equiv \sin \theta_{12}$. By using (37) and (38), one finds the values of a and b from the input, with a twofold ambiguity only. Then θ' is given by

$$c'^2 - s'^2 = \frac{1}{m_1^2 m_2^2 (M_1^2 - M_2^2)} \left(\left| s_{12}^2 m_1 + c_{12}^2 m_2 e^{i\Delta} \right|^2 a^4 - \left| c_{12}^2 m_1 + s_{12}^2 m_2 e^{i\Delta} \right|^2 b^4 \right). \quad (39)$$

With a and b known, \mathcal{Q} is found as a function of the input by use of

$$H_{11} = a^2 c'^2 + b^2 s'^2, \quad (40)$$

$$\text{Im}[(H_{12})^2] = (b^2 - a^2)^2 \frac{M_1 M_2}{M_2^2 - M_1^2} \frac{m_2^2 - m_1^2}{m_1 m_2} c_{12}^2 s_{12}^2 \sin \Delta. \quad (41)$$

Equations (37)–(41) are identical with those of the \mathbb{Z}_2 model, derived in [12]. In order to compute the baryon-to-photon ratio of the Universe, η_B , one must [12] multiply \mathcal{Q} by (i) $M_1/(10^{11} \text{ GeV})$, (ii) a numerical factor of order 10^{-9} , (iii) a function of M_2/M_1 , and (iv) $(\ln K_1)^{-3/5}$, where $K_1 \propto H_{11}/M_1$. (All these factors are given and explained in [12], together with references to the original papers.) One may then compute η_B as a function of the input.

Most crucial is the behaviour of η_B as a function of m_1 when $m_2^2 - m_1^2 = \Delta m_\odot^2$ is kept fixed. One finds that η_B grows with m_1 , finding a maximum for $m_1 \sim 4 \times 10^{-3} \text{ eV}$, afterwards decreasing rapidly for a larger m_1 . Now, the present model—contrary to what happened in the model treated in [12], wherein m_1 was free—has $m_3 = 0$ and, therefore, $m_1 \simeq \sqrt{\Delta m_{\text{atm}}^2} \sim 0.05 \text{ eV}$. For such a high value of m_1 the baryon-to-photon ratio turns out to be hopelessly small. Thus, in the present model, contrary to what happened in the \mathbb{Z}_2 model [10] worked out in [12], leptogenesis is not a viable option for explaining the baryon asymmetry of the Universe.

5 Conclusions

In this letter we have discussed an extension of the lepton sector of the Standard Model with two right-handed neutrino singlets and the seesaw mechanism. The model, which was originally proposed in [21], is based on the lepton number $\bar{L} = L_e - L_\mu - L_\tau$. Zeros in the 2×3 neutrino Dirac mass matrix are enforced by \bar{L} invariance, and as a consequence the model features the predictions $\theta_{13} = 0$ and a hierarchical neutrino mass spectrum with $m_3 = 0$.⁷ The lepton number \bar{L} is softly broken in the 2×2 Majorana mass matrix M_R of the right-handed neutrino singlets, by the two entries R and S in (13) which would be zero in the case of exact \bar{L} invariance. One obtains $\Delta m_\odot^2 \neq 0$ and $\theta_{12} \neq 45^\circ$ from that soft breaking. However, $\theta_{12} \sim 33^\circ$ requires the soft breaking to be ‘strong’, which means that R and S are of the same order of magnitude as the element M in M_R which is allowed by \bar{L} invariance. Thus the model discussed here has the property that in M_R there is no trace of \bar{L} invariance, whereas the form of the Dirac mass matrix is completely determined by that invariance.

We have argued that, for models with one Higgs doublet like the present one, the configuration $m_3 = 0$ together with $\theta_{13} = 0$ is stable under the renormalization-group evolution.

A further prediction of our model is the range for the effective mass in neutrinoless $\beta\beta$ decay, in particular the lower bound given by (30); the order of magnitude of that effective mass is the square root of the atmospheric mass-squared difference.

Since there is only one CP -violating phase in our model, we have also considered the possibility of thermal leptogenesis; however, it turns out that this mechanism is unable to generate a realistic baryon asymmetry of the Universe. This is because in our model the neutrino mass m_1 is too large, due to the inverted mass hierarchy.

In summary, we have shown by way of a very economical example that—contrary to claims in the literature—models based on the lepton number $L_e - L_\mu - L_\tau$ are not necessarily incompatible with the solar mixing angle being significantly smaller than 45° .

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⁷These predictions are common with other models based on \bar{L} invariance [27].

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